LECTURER1

Limits and continuity

Function

Defenition:A function f is a rule that assigns to each element x in a set A exactly one element, called f(x), in a set B

Domain and Range of a Functions

Recall the definition of a function.

Let $f: A \rightarrow B$ be a function from A to B. The set A is called the domain of f and the set B is called the codomain of f. The set $f(A) = \{f(x) \setminus x \in A\}$ is called the range of f

For example

Let A={1,2,3,4}and B={v,w,x,y,z} ,let $f: A \rightarrow B$ be f={(1,w),(2,y),(3,y),(4,z)} then

(a)The domain of f is {1,2,3,4}

(b)The codomain of f is (v,w,x,y,z}

©The range of f is f(A)={f(1),f(2),f(3),f(4)}={w,y,y,z}={w,y,z}

<u>Definition</u>

A relation is a function if and only if each object in its domain is paired with one and only one object in its range. This is not an easy definition, so let’s take our time and consider a few examples.

Ex(1)
Consider the relation R .

R = (0, 1),(0, 2),(3, 4) The domain is {0, 3} and the range is {1, 2, 4}. Note that the number 0 in the domain of R is paired with two numbers from the range, namely, 1 and 2. Therefore, R is not a function. There is a construct, called a mapping diagram, which can be helpful in determining whether a relation is a function.

To craft a mapping diagram, first list the domain on the left, then the range on the right, then use arrows to indicate the ordered pairs in your relation, as shown in Figure 1 .

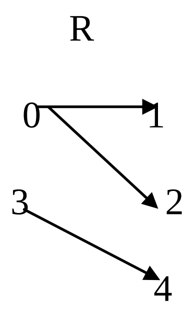


Figure 1. A mapping diagram for R.

It’s clear from the mapping diagram in Figure 1 that the number 0 in the domain is being paired (mapped) with two different range objects, namely, 1 and 2. Thus, R is not a function. Let’s look at another example.

Example(2)

Is the relation described in T a function? First, the listing of the relation T.

T = (1, 2),(3, 2),(4, 5) Next, construct a mapping diagram for the relation T. List the domain on the left, the range on the right, then use arrows to indicate the pairings, as shown in Figure2

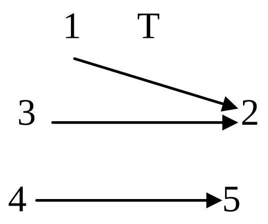


Figure 2. A mapping diagram for T

LECTURER2

From the mapping diagram in Figure 2, we can see that each domain object on the left is paired (mapped) with exactly one range object on the right. Hence, the relation T is a function

Functions and different types of functions

A relation is a function if for every x in the domain there is exactly one y in the codomain. A vertical line through any element of the domain should intersect the graph of the function exactly once. (one to one or many to one but not all the Bs have to be busy) A function **is injective** if for every y in the codomain B there is at most one x in the domain.

A horizontal line should intersect the graph of the function at most once (i.e.not at all or once). (one to one only but not all the Bs have to be busy) A function **is bijective** if for every y in the codomain there is exactly one x in the domain.

A horizontal line through any element of the range should intersect the graph of the function exactly once. (one to one only and all the Bs must be busy). A function **is surjective** if for every y in the codomain B there is at least one x in the domain. A horizontal line intersects the graph of the function at least once (i.e.once or more). The range and the codomain are identical. (one to one or many to one and all the Bs must be busy)

Example: Find the Domain and the Range for the function $f(x) = \frac{1}{x-1}$

D: $x \neq 1$ R: $y \neq 0$

H.W.

Find the Domain and the Range for the following functions

(1) $y = \sqrt{x^2 - 4x + 3}$

(2) $y = x^2$

LECTURER3

The limits

Assume f is defined in a neighborhood of c and let c and L be real numbers. The function f has limit L as x approaches c if, given any positive number ε, there is a positive number d δ such that for all x, $0 \prec |x-c| \prec \delta \Rightarrow |f(x)-L| \prec \varepsilon$

We write $\lim\limits_{x \to c} f(x) = L$

The sentence limx→c fx L is read, “The limit of f of x as x approaches c equals L.” The notation means that the values f (x) of the function f approach or equal L as the values of x approach (but do not equal) c.

Ex(3):

If $f(x) = \dfrac{x^2-3x+2}{x-2}, x \neq 2$ find $\underset{x \to 2}{Lim} f(x)$

Solution: $\underset{x \to 2}{Lim}\, f(x) = \underset{x \to 2}{Lim} \dfrac{x^2-3x+2}{x-2} = \underset{x \to 2}{Lim} \dfrac{(x-2)(x-1)}{(x-2)} = \underset{x \to 2}{Lim}(x-2) = 2-1=1$

Limits at Infinity

We note that when the limit of a function f(x) exist as approachs infinity, we write $\underset{x \to \infty}{Lim}\, f(x) = L$

Also, we write $\underset{x \to +\infty}{Lim}\, f(x) = L$ for +ive values of x and $\underset{x \to -\infty}{Lim}\, f(x) = L$ for

-ive values of x

For one sided and Tow sided limits,we have $\underset{x \to \infty}{Lim}\, f(x) = L$ iff $\underset{x \to +\infty}{Lim}\, f(x) = L$ and $\underset{x \to -\infty}{Lim}\, f(x) = L$

LECTURER4

Some obvious Limits

(1)If k is constant,then $\lim_{x\to+\infty} f(x)=k$ and $\lim_{x\to-\infty} f(x)=k$

(2) $\lim_{x\to\infty}\frac{1}{x}=0$, $\lim_{x\to+\infty}\frac{1}{x}=0$,and) $\lim_{x\to-\infty}\frac{1}{x}=0$

(3)) $\lim_{x\to0}\frac{1}{x}=\infty$, (3)) $\lim_{x\to0+}\frac{1}{x}=\infty$,and (3)) $\lim_{x\to0-}\frac{1}{x}=\infty$

Examples:

Find the following limits

(1) $\lim_{x\to\infty}\frac{x}{2x+3}=\lim_{x\to\infty}\frac{1}{2+\frac{3}{x}}=\frac{1}{2}$

هنا نضرب ونقسم في مرافق المقدار لايجاد قيمة الغاية

(2) $\lim_{x\to\infty}(\sqrt{x^2+1}-x=\lim_{x\to\infty}(\sqrt{x^2+1}-x.\frac{\sqrt{x^2+1}+x}{(\sqrt{\sqrt{x^2+1}+x}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x}}+1}=\frac{0}{2}=0$

LECTURER4

Theorems on limits(Calculation Technique)

1-Uniqueness of limit

If $\lim_{x\to a} f(x) = L$ then L is unique

2-Limit of constant

If f(x)=c, where c is constant then $\lim_{x\to a} f(x) = \lim_{x\to a} c = c$

3-Obvious limit

If f(x)=x then $\lim_{x\to a} f(x) = \lim_{x\to a} x = a$

4-Limit of Som

If $f(x) = f_1(x) \pm f_1(x) \pm f3(x) \pm \ldots\ldots \pm f_n(x)$ and $\lim_{x\to a} f_i(x) = L_i$,i=1,2,….,n,then

$$\lim_{x\to a} f(x) = \lim_{x\to a} f_1(x) \pm \lim_{x\to a} f_2(x) \pm \ldots \lim_{x\to a} f_n(x) = L_1 \pm L_2 \pm \ldots\ldots \pm L_N$$

5-Limit of product

If $f(x) = f_1(x).f_2(x).\ldots\ldots f_n(x)$ and $\lim_{x\to a} f_i(x) = L_i$, i=1,2,….,n,then $\lim_{x\to a} f(x) =$ $\lim_{x\to a} f_1(x).\lim_{x\to a} f_2(x).\ldots\ldots \lim_{x\to a} f_n(x). = L_1.L_2.\ldots\ldots L_n$

6-Limit of Quotient

If $f(x) = \dfrac{g(x)}{h(x)}$ and $\lim_{x\to a} g(x) = L_1$,and $\lim_{x\to a} h(x) = L_2$, $L_2 \neq 0$ then $\lim_{x\to a} f(x) =$

$$\lim_{x\to a} f(x) = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{L_1}{L_2}$$

EXAMPLE 3 Using Properties of Limits

Use the observations $\lim_{x\to c} k = k$ and $\lim_{x\to c} x = c$, and the properties of limits to find the following limits.

(a) $\lim\limits_{x\to c} (x^3 + 4x^2 - 3)$ (b) $\lim\limits_{x\to c} \dfrac{x^4 + x^2 - 1}{x^2 + 5}$

SOLUTION

(a) $\lim\limits_{x\to c} (x^3 + 4x^2 - 3) = \lim\limits_{x\to c} x^3 + \lim\limits_{x\to c} 4x^2 - \lim\limits_{x\to c} 3$ Sum and Difference Rules

$= c^3 + 4c^2 - 3$ Product and Constant Multiple Rules

(b) $\lim\limits_{x\to c} \dfrac{x^4 + x^2 - 1}{x^2 + 5} = \dfrac{\lim\limits_{x\to c} (x^4 + x^2 - 1)}{\lim\limits_{x\to c} (x^2 + 5)}$ Quotient Rule

LECTURER5

DERIVATIVE RULES

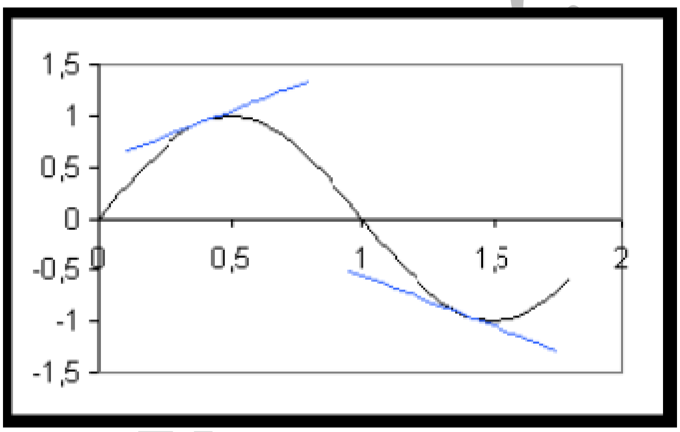
Definition

The derivative of a function f at a point x, written $f'(x)$, is given by:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

if this limit exists.

Graphically, the derivative of a function corresponds to **the slope of its tangent line at one specific point.** The following illustration allows us to visualise the tangent line (in blue) of a given function at two distinct points. Note that the slope of the tangent line varies from one point to the next. The value of the derivative of a function therefore depends on the point in which we decide to evaluate it. By abuse of language, we often speak of the slope of the function instead of the slope of its tangent line.



Notation

Here, we represent the derivative of a function by a prime symbol. For example, writing $f'(x)$ represents the derivative of the function f evaluated at point x. Similarly, writing $(3x + 2)'$ indicates we are carrying out the derivative of the function $3x + 2$. The prime symbol disappears as soon as the derivative has been calculated.

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}\left(a^x\right)=\ln a\cdot a^x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right)=f(x)\cdot g'(x)+g(x)\cdot f'(x)$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}(\arcsin x)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right)=f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}(\operatorname{arcsec} x)=\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\ln x)=\frac{1}{x}$$

$$\frac{d}{dx}(\sinh x)=\cosh x$$

$$\frac{d}{dx}(\cosh x)=\sinh x$$